A fully integrated roll deflection and strip buckling model for calculating strip flatness

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Strip flatness is an important quality aspect of cold rolled strip. In a tandem cold mill, latent strip flatness errors depend on many factors, such as work roll profiles, rolling forces, bending forces, and thermal expansion, in a complicated interaction. If latent flatness errors manifest themselves, they may lead to centre buckles, wavy edges, quarter buckles or other flatness defects. Excessive strip buckling may even cause operational problems in the mill such as pinches and work roll damage, resulting in downtime and high roll repairing costs. In this paper a fully integrated roll deflection and strip buckling numerical model is presented. The model comprises a strip rolling model, a roll stack deflection model, a roll flattening model and finally a strip buckling model. The roll stack deflection model contains the work roll and backup roll profiles and the main deformation aspects of the rolls: bending, Poisson effect and flattening. This allows us to calculate the non-uniform thickness reduction over the strip width. Non-uniformity of the thickness reduction translates into flatness errors, and these are input to the buckling model to evaluate when these flatness errors become manifest. The model is illustrated for a practical case study of a 4-high tandem rolling mill.

KEYWORDS: ROLLING, MODELS, STRIP FLATNESS, SHAPE, NUMERICAL METHODS

INTRODUCTION

The quality of the steel strip depends on both its material and geometrical properties. Inhomogeneities in strip thickness across the width direction (profile) lead to longitudinal internal stresses (shape). Flatness issues arise when shape manifests itself due to insufficient strip geometrical stiffness (buckling). Flatness can be studied as an elastic buckling problem, to not only get the critical buckling limit P_{cr} (imposed load multiplied by the 1st eigenvalue), but also the way the strip will buckle (eigenvector). Note that the magnitude of the buckled shape cannot be obtained by the elastic buckling analysis. The latter would require highly complex post-buckling analysis.

Strip flatness is an important quality aspect of cold rolled strip. Flatness errors depend on many factors, such as work roll profiles, rolling forces, bending forces, and thermal expansion. The interaction of all these components lead to inhomogeneous reduction across the strip width. At excessive internal stresses, the transversal cross-

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section of the strip has regions where the normal stresses may alternate between positive (tension) and negative (compression) values. Depending on the distribution of the regions under tension/compression, different forms of buckled geometries may appear. Most typical flatness errors include centre buckles, wavy edges and quarter buckles.

Excessive strip buckling may even cause operational problems in the mill such as pinches. These pinches can cause work roll damage, resulting in downtime and high roll repairing costs. In this paper a fully integrated roll deflection and strip buckling model is presented, together with a case study of a 4-high mill. In [1] strip deformation models and buckling models have been coupled but based on a finite element approach. In this work the models are based on finite differences, which are typically faster to compute and therefore more suited to use for fast analysing tools.

The outline of the paper is as follows. First the roll stack deflection model and buckling model will be explained briefly. Then validation of important buckling modes is presented, followed by a practical example from a 4-high rolling mill.

ROLL STACK DEFLECTION MODEL

The roll stack model is a segmented model dividing the rolls into discs. The model is similar to the ones described in e.g. [2]. Inputs are the roll geometry including the roll profiles, the total roll force, and the bending forces. The output is the deformed roll gap profile and strip shape at the entry and exit. This strip shape is used in the buckling model.

All constitutive equations are written out for each segment and then are linearised around the current working point. The linearised set of equations is a matrix equation $\mathbf{M}\mathbf{x}=\mathbf{l}$ where matrix $\mathbf{M}$ contains the coefficients of the system of equations, vector $\mathbf{x}$ contains the unknowns, and vector $\mathbf{l}$ contains the constant terms. Solving this matrix equation gives a new working point, linearise again etc. until the difference in subsequent solutions becomes benign.

An example of a coefficient matrix $\mathbf{M}$ and its corresponding unknown vector $\mathbf{x}$ for a symmetric 4-high mill is shown in Fig. 1. Submatrices $\mathbf{M}_{WR}$ and $\mathbf{M}_{BUR}$ correspond to the work roll and back-up roll, respectively. These matrices describe the deformation of the rolls (e.g. curvature-bending relation, moment balance, force balance, and Poisson effect). Submatrix $\mathbf{M}_{WR\text{-}BUR}$ describes the flattening relation on the interfaces between work roll and back-up roll. Submatrix $\mathbf{M}_{RG}$ describes the relation between the roll force distribution, the strip and the deformation of the work rolls in the roll gap. Submatrices $\mathbf{x}_{WR}$ and $\mathbf{x}_{BUR}$ contain the unknowns: curvature ($\partial^2 v/\partial x^2$), gradient ($\partial v/\partial x$), deflection ($v$), moment ($M$), shear force ($D$), normal force ($F$) and line load (q). Sub-vector $\mathbf{x}_{Strip}$ contains the unknowns: strip profile, strip shape, and entry tension.

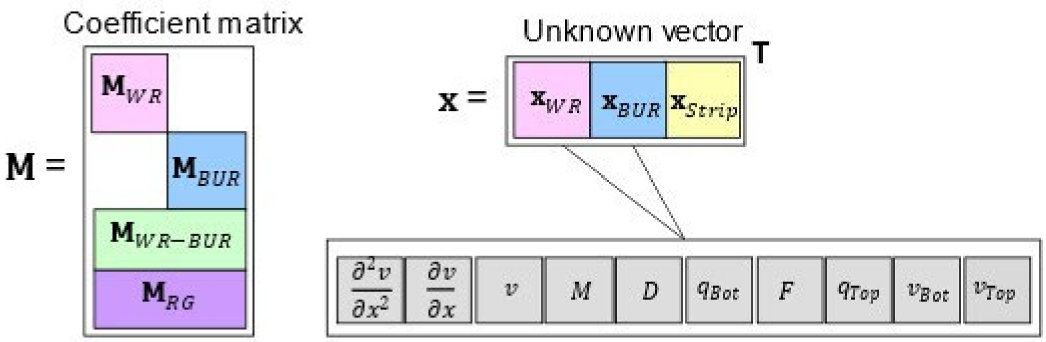

Fig.1 - Coefficient matrix $\mathbf{M}$ and its corresponding unknown vector $\mathbf{x}$ for a symmetric 4-High mill.

BUCKLING MODEL

The buckling model is a finite difference model based on the Kirchhoff's thin plate bending theory, see [3]. The governing equation in Cartesian coordinates is given by

$$\nabla^4 w = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} - 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}\right) \tag{1}$$

where ∇^4 is the bi-Laplacian operator, w is the deflection, D is the flexural rigidity given by $D=(Eh^3)/12/(1-\nu^2)$, E is the Young's modulus, ν is the Poisson ratio and h is the strip thickness. x is the coordinate in the width direction and y is the coordinate in the rolling direction. The in-plane specific forces N_x, N_y, and N_{xy} can be found from the solution of the plane stress problem for the given geometry (rectangular plate), in-plane external loadings (line tension), and internal stresses (shape).

The buckling limit is obtained by transforming the above problem into an eigenvalue problem. Here, a multiplication factor λ is introduced such that $N_x = -\lambda N_x$, $N_y = -\lambda N_y$, $N_x = -\lambda N_{xy}$, where $\bar{N}_x$, $\bar{N}_y$, and $\bar{N}_{xy}$ are the reference value of the loads. Substituting these into (1) results into an alternative form of the governing equation:

$$\nabla^4 w = -\frac{\lambda}{D}\left(\bar{N}_x \frac{\partial^2 w}{\partial x^2} - 2\bar{N}_{xy}\frac{\partial^2 w}{\partial x \partial y} + \bar{N}_y \frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

The solution of the above problem requires two boundary conditions to be satisfied at each side of the strip:

- clamped $(w = 0$ and $\partial w/\partial x = 0$ (or $\partial w/\partial y = 0$))
- simply supported $(w = 0$ and $\partial^2 w/\partial x^2 = 0$ (or $\partial^2 w/\partial y^2 = 0$)) and
- free edge $(\partial^2 w/\partial y^2 + \nu\, \partial^2 w/\partial x^2 = 0$ and $\partial\left[\frac{\partial^2 w}{\partial y^2} + (2-\nu)\frac{\partial^2 w}{\partial x^2}\right]/\partial y = 0$ (or $\partial\left[\frac{\partial^2 w}{\partial x^2} + (2-\nu)\frac{\partial^2 w}{\partial y^2}\right]/\partial x = 0$)).

In combination with a finite difference method, (2) can be written as

$$\mathbf{A}\mathbf{w}_i = \lambda_i \mathbf{B}\mathbf{w}_i \tag{3}$$

for $i=1,\dots,n$ where n is the size of the matrices A and B. The matrix equation shown in (3) is known as the generalized eigenvalue problem. Here, λ_i are the eigenvalues and w_i are the corresponding eigenvectors. The 1st eigenvalue and its eigenvector are used to determine the critical buckling limit, P_{cr}, and the way the strip will buckle, respectively.

ELASTIC BUCKLING CALCULATIONS VALIDATION

Simulations settings

The aim of this chapter is to evaluate the accuracy of integrated model in predicting different buckling modes for strips under residual stresses. The validation is done at various strip buckling cases in two ways:

- against the paper by Fischer [4], where buckling analysis of cold rolled strip is studied without coupling to a rolling model. These are cases 1-6, see Fig. 2 for the numbering,
- against the commercial finite elements software (ANSYS, 2020 R2), cases 7-13.

All 13 simulations share the same strip width, thickness, and elasticity properties. At the first 6 cases the strip length is chosen to be 4 times the wavelength as reported by Fischer. The reason for using a precise multiple is that exact results are then obtained as if a strip of infinite length is used. At the rest of the cases, the chosen strip length is four times the strip width, sufficiently long not to affect the results. The short edges of the strip are either simply supported (cases 1-12) or clamped (case 13).

The side edges are free. The mesh in both the integrated model and in ANSYS consist of 100 and 200 elements in the width and rolling direction, respectively. The coordinate system is such that x is in the strip width direction and y in the length (rolling) direction.

There are various ways to mathematically describe the distribution of the residual stresses along the strip width direction. Fischer's paper uses both the cosine and the polynomial description (cases 1-6), whereas at cases 8-13 the W-polynomials were used. A schematic of the input residual stress per case is illustrated in Fig. 2 (left part). The red/green colour represent the areas under compression/tension.

Simulations results

The most important result of the elastic buckling analysis is the 1st eigenvalue (and eigenvector) because it is connected to the critical buckling load. Focusing on centre buckles (cases 1-6 & 8), the critical load difference between the developed model and Fischer's work is very small for the first 5 cases, i.e. <0.4%. In the 6th and 8th case the deviation is 3.3% and 1.0%, respectively, and it may be attributed to mesh size difference. Nevertheless, the results are satisfactory for buckling prediction in the rolling process, since an uncertainty of about 3% in calculating critical buckling loads is acceptable, in view of other uncertainties when calculating the process conditions in rolling mills. The wavy edges cases (7, 9-11) and the quarter buckles ones (12-13), exhibit an eigenvalue error of less than 1.5% between the developed model and ANSYS. Concerning the eigenvectors, the long wavelength (case 9) is simulated with remarkable accuracy. However, when the wavelength is shorter (cases 7,10,11), minor deviations between the buckling model and the FEA eigenvectors were observed, with a maximum error in wavelength of 2.8%. The same also holds for the quarter buckles, which both have short wavelength.

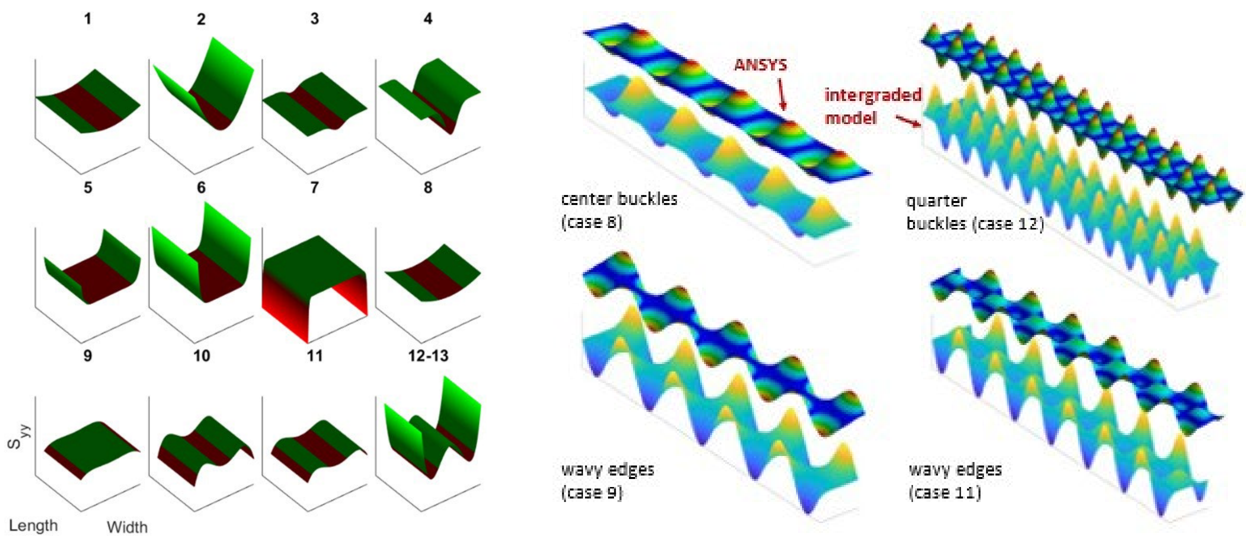

Fig.2 - Left: Schematic of the longitudinal stress per case (red = compression, green = tension). Right: Eigenvector prediction of cases 8, 9, 11, 12 by the integrated model versus ANSYS.

As stated in the previous paragraph, the integrated model exhibits eigenvector results similar to Fisher and ANSYS. In principle, a structural component prone to buckling will follow the equilibrium path of the 1st buckling mode, being the lowest one. However, in most of the investigated cases in this report, the first 2 eigenvalues are very close to each other (<1% difference). This has an impact on the analysis. Small changes at the mesh or at the distribution of the residual stresses may lead the solution to follow the path of a higher buckling mode. These modes are very similar, and in most cases symmetric to each other. The eigenvalue calculation is more sensitive to numerical errors, if the first few eigenvectors are almost in the same direction of the vector space.

In conclusion, the integrated model is deemed capable of modelling the elastic buckling of strip under residual stresses. Being a finite difference model, it outperforms ANSYS in calculation speed and can thus be used for both on/off-line calculations.

WORK ROLL BENDING AND STRIP SHAPE

Strip shape is strongly affected by the work roll bending forces, $\mathrm{BF_{WR}}$. Excessive roll stack bending leads to larger thickness reduction (equivalent to strip elongation at plane strain) at either strip centre ($\mathrm{BF_{WR}}>0$) or edges ($\mathrm{BF_{WR}}<0$). The strip being a single entity, adjacent regions with different reductions will restrict the variations in elongation. This will generate internal stresses (strip shape) in the rolling direction.

In this paragraph the integrated model is used at a case-study of the work roll bending force effect on strip shape. A 4-high tandem mill with typical roll diameters has been used. The strip is 2000 mm wide. The entry/exit line tension usually prevents strip shape issues to manifest themselves in the line. Therefore, significant positive/negative bending force was used to generate sufficient internal longitudinal compressive stresses to overcome the tensile line tension. The resulting longitudinal stresses together with corresponding buckled strip are shown in Fig. 3. The strip runs in the figure from left to right, as indicated by the arrow. The colour of the strip indicates the longitudinal stress, with blue being compressive and yellow tensile and green in-between. Note that the amplitude of the buckles is exaggerated at the graph. On the left, centre buckles arise due to positive bending, whereas on the right wavy edges are demonstrated for negative bending.

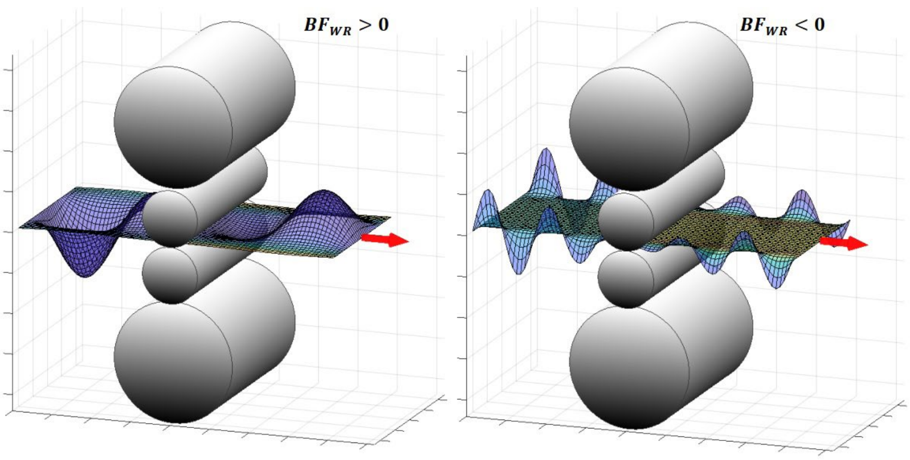

Fig.3 - 3-D visualisation of the roll stack with manifest strip shape (blue = compression, yellow= tension). The deformation of the roll stack is not visible in the graph.

CONCLUSIONS

The presented model combines roll stack deflection and buckling analysis to predict latent and manifest shape errors during rolling. It links the most common process parameters directly with the buckling analysis results, providing an intuitive simulation tool. The elastic buckling validation shows that errors are below 3.3% and 2.8% for the critical loads and wavelengths, respectively. The application section demonstrates that the buckling model is capable of predicting both critical buckling limit and modes that are characteristic for rolling.

REFERENCES

[1] A. Sami, R. Nakhoul, H. Zahrouni, P. Montmitonnet and N. Legrand, "Applications of advanced models to prediction of flatness defects in cold rolling of thin strips," in 15th International Conference on "Advances in Materials and Processing Technologies - AMPT 2012, Wollongong, 2012.

[2] S.-H. Kim, G.-H. Lee, S.-J. Song and B.-M. Lee, "Study on the improved accuracy of strip profile using numerical formula model," Journal of Mechanical Science and Technology, vol. 25, no. 8, pp. 2101-2109, 2011.

[3] E. Ventsel and T. Krauthammer, Thin Plates and Shells, Marcel Dekker, 2001.

[4] F. D. Fisher, F. G. Rammerstorfer and N. Friedl, "Residual Stress-Induced Center Wave Buckling of Rolled Strip Metal," ASMA, vol. 70, pp. 84-90, 2003.

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